The generalized formulae reduce to the familiar ones associated with the more common grounded vertical antenna when $\theta_1 = 0$. Thus, for this case

$$
(13)\quad \begin{cases}
A_1 = \dfrac{I_m}{4\pi r} e^{-jk[r - S\sin\theta\cos(\varphi-\beta)]}\left\{\dfrac{e^{jkl\cos\theta} - \cos kl - j\sin kl\cos\theta}{k\sin^2\theta}\right\} \\[2ex]
A_2 = \dfrac{I_m}{4\pi r} e^{-jk[r - S\sin\theta\cos(\varphi-\beta)]}\left\{\dfrac{e^{-jkl\cos\theta} - \cos kl + j\sin kl\cos\theta}{k\sin^2\theta}\right\} \\[2ex]
E_\theta = \dfrac{j\omega\mu I_m}{2\pi r k} e^{-jk[r - S\sin\theta\cos(\varphi-\beta)]}\left\{\dfrac{\cos(kl\cos\theta) - \cos kl}{\sin\theta}\right\} \\
E_\varphi = 0
\end{cases}
$$

Radiation Pattern

In order to study the radiation pattern of a slant antenna and its image it is easier to rotate and translate the axis so that the antenna lies at the origin in the y-z plane - i.e. $S = 0$, $\varphi_1 = \frac{\pi}{2}$. It is obvious that so far as considering the radiation pattern from a single antenna no loss in generality results from this shift and rotation of the axis. For these conditions (5), (7), and (11) become

$$
(14)\quad \begin{cases}
A_1 = \dfrac{I_m e^{-jkr}}{4\pi r k}\left\{\dfrac{e^{jkl\cos\delta_1} - \cos kl - j\sin kl\cos\delta_1}{\sin^2\delta_1}\right\} \\[2ex]
A_2 = \dfrac{I_m e^{-jkr}}{4\pi r k}\left\{\dfrac{e^{-jkl\cos\delta_2} - \cos kl + j\sin kl\cos\delta_2}{\sin^2\delta_2}\right\} \\[2ex]
\cos\delta_1 = \sin\theta\sin\theta_1\sin\varphi + \cos\theta\cos\theta_1 \\
\cos\delta_2 = -\sin\theta\sin\theta_1\sin\varphi + \cos\theta\cos\theta_1 \\
E_\theta = j\omega\mu\left[(A_1 + A_2)\cos\theta_1\sin\theta - (A_1 - A_2)\sin\theta_1\cos\theta\sin\varphi\right] \\
E_\varphi = j\omega\mu(A_1 - A_2)\sin\theta_1\cos\varphi
\end{cases}
$$

The vector potentials A_1 and A_2 have the same directions as the activating currents I_1 and I_2, respectively, and the resultant vector potential A is the vector sum of A_1 and A_2. (See Fig. 3). A, A_1, and A_2 lie in the y - z plane.

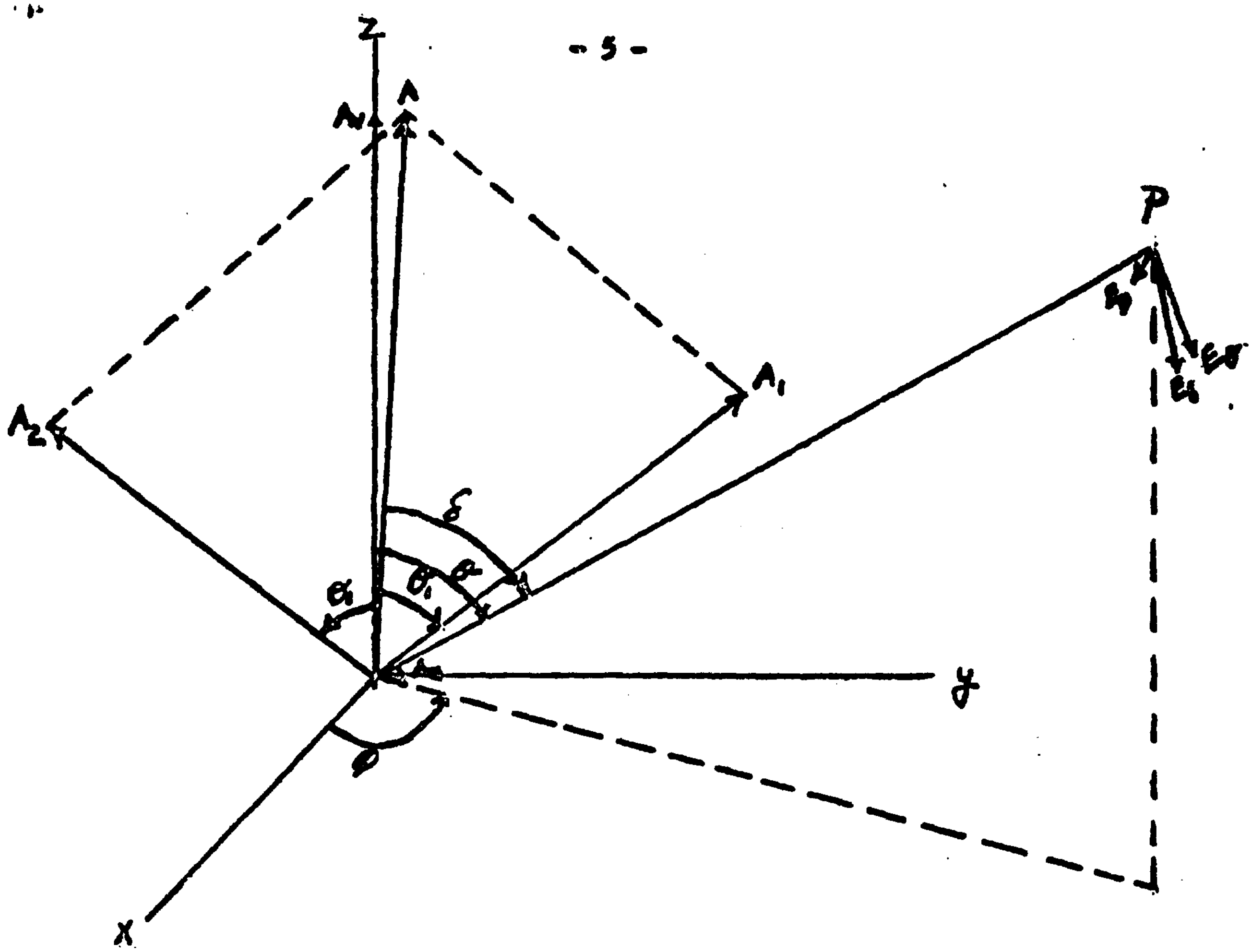

Fig. 3

The usual method of determining the directions of maximum and minimum radiation by setting the derivatives equal to zero are too cumbersome and unmanageable so that an indirect approach is necessary.

It is apparent from (9) that E_δ will be zero if δ is equal to zero.

From Figure 3 it is observed that this null direction must lie in the y - z plane, if such a null exists. Furthermore, it is noted from (5) and (8) that the contribution of A_1 to E_δ is zero for O P in the I_1 direction and similarly the contribution of A_2 to E_δ is zero for O P in the I_2 direction. Consequently, the directions for minimum E_δ should lie in the y - z plane between the I_1 and I_2 directions.

Another approach is to consider the magnetic fields H_1 and H_2 which are set up by magnetic vectors A_1 and A_2, respectively. Only in the y - z plane between the directions of I_1 and I_2 will these vector magnetic fields H_1 and H_2 be in exactly opposite directions (along the x-axis). It seems reasonable from this approach to expect that the minimum resultant field H will lie in the y - z plane between the directions of I_1 and I_2.

Probably the most practical approach to determining the exact direction of minimum radiation is to plot the radiation for this y - z plane between the directions of I_1 and I_2 and so determine it graphically.

For the y - z plane the field intensity and vector potentials reduce to

$$
(15)\left\{
\begin{aligned}
A_1 &= \frac{I_m e^{-jkr}}{4\pi r k}\left[\frac{e^{jkl\cos(\theta-\theta_1)} - \cos kl - j\sin kl\cos(\theta-\theta_1)}{\sin^2(\theta-\theta_1)}\right] \\
A_2 &= \frac{I_m e^{-jkr}}{4\pi r k}\left[\frac{e^{-jkl\cos(\theta+\theta_1)} - \cos kl + j\sin kl\cos(\theta+\theta_1)}{\sin^2(\theta+\theta_1)}\right] \\
E_\theta &= j\omega\mu\left[-A_1\sin(\theta_1-\theta) + A_2\sin(\theta_1+\theta)\right] \\
E_\varphi &= 0
\end{aligned}
\right.
$$

The value of θ in (15) may be plus or minus in order to cover the entire y - z plane. In this two coordinate system the limiting directions for minimum radiation may be expressed as:

$$(16)\qquad -\theta_1 \leq \theta \leq \theta_1$$

Again considering the vector magnetic field intensities H_1 and H_2 it is apparent that only in the directions perpendicular to the y - z plane will these field intensities be in exactly the same space phase (in the y direction). Thus, the maximum radiation occurs for

$$
(17)\begin{cases}
\varphi = 0 \text{ or } \pi \\
\theta = \frac{\pi}{2} \\
E_\theta = \frac{j\omega\mu I_m e^{-jkr}}{2\pi r k}(1-\cos kl) \\
E_\varphi = 0
\end{cases}
$$

In treating an array of slant antennas the most practical approach usually is to calculate the resultant vertical and horizontal vector potentials and then obtain the resultant fields from (11).

Summary

This report is a brief study of the radiation characteristics of a grounded slant antenna. The general formulae for radiation from this type of antenna over a perfectly conducting earth are derived for sinusoidal current distribution.

Harry Fine